

# Time evolution of rolling tachyons for a brane-antibrane pair

Haewon Lee<sup>1</sup> and W. S. l'Yi<sup>2</sup>

*Department of Physics*  
*Chungbuk National University*  
*Cheongju, Chungbuk 361-763, Korea*

## Abstract

A precise form of the time evolution of rolling tachyons corresponding to a brane-antibrane pair is investigated by solving the Hamiltonian equations of motion under the assumption that in a region far away from branes the tachyon vacuum is almost already achieved, even at the beginning of the rolling.

Oct. 23, 2002

---

<sup>1</sup>E-mail address: [hwlee@hep.chungbuk.ac.kr](mailto:hwlee@hep.chungbuk.ac.kr)

<sup>2</sup>E-mail address: [wslyi@hep.chungbuk.ac.kr](mailto:wslyi@hep.chungbuk.ac.kr)

# 1 Introduction

After the discovery of tachyon condensation[1] of non-BPS D-branes or brane-antibrane pairs, much studies have been done to investigate the physics of unstable branes[2]. Some possible roles of rolling tachyons on cosmology in relation to inflation are also discussed by many authors[3]. Recently Sen proposed a new way to quantum gravity by interpreting tachyon field as world lines of non-interacting, non-rotating dust[4]. The important ingredient of his proposal is that after sufficient rolling down of tachyon to vacuum, tachyon field is almost the same as the time, thus making it possible to identify the elusive intrinsic time in general gravity with tachyon field.

In this paper we use the Hamiltonian formalism[5] while solving the equations of motion to obtain a precise form of time evolution of tachyon field. We first assume homogeneous tachyon configuration in space, and then generalize it to a more realistic brane-antibrane configuration. In this case we take the ansatz that near the vacuum tachyon field  $T$  can be written as a summation of a function of time and a linear function of one of the spatial coordinate  $x$ . It will be proved in section 3 that this kind of ansatz is valid only if the tachyon potential around the vacuum looks like  $V(T) = \exp(-\frac{1}{2}\alpha T)$  with some constant  $\alpha$ . This is consistent with string theoretic calculations that the tachyon potential should indeed be of that form[6]. The potential around the region where D-branes reside is more complicated, and this fact is related to the fact that it is not easy to guess any specific form of the potential around  $T \approx 0$ . But as long as one is far from the branes, one is able to have a precise form of the time evolution of  $T$ .

In section 2, we briefly introduce the Hamiltonian formalism for the usual tachyon Lagrangian. The canonical equations of motion are solved for homogeneous field configuration. This way of solving the equations of motion is more intuitive and readily applicable to more realistic systems such as brane-antibrane pairs than the Lagrangian formalism[7]. In section 3, we assume a specific ansatz for tachyon field corresponding to a pair of brane-antibrane. By solving the Hamiltonian equation of motion we obtain a

precise form of time evolution of tachyon around the vacuum.

## 2 Tachyon action and the equation of motion

The Lagrangian density for tachyon field  $T(x)$  on a space-filling D-brane system in a flat  $p + 1$  dimensional space is given by

$$\mathcal{L} = -V(T)\sqrt{1 + \eta^{\mu\nu}\partial_\mu T\partial_\nu T}, \quad (1)$$

where  $\eta^{\mu\nu} = \text{diag}(-1, 1, \dots, 1)$ , and we use the convention  $c = \hbar = \alpha' = 1$ . The tachyon potential  $V(T)$  is known to be maximum at  $T = 0$ , and vanishes as  $T$  goes to infinity. Instead of using the usual Lagrangian formalism for solving the equations of motion, we choose the Hamiltonian formalism[5] which we briefly summarize.

The canonical momentum density  $\Pi(x)$  corresponding to  $T$  is given by

$$\Pi(x) = \frac{\partial\mathcal{L}}{\partial(\partial_0 T(x))} = \frac{V(T)\partial_0 T}{\sqrt{1 - (\partial_0 T)^2 + (\nabla T)^2}}. \quad (2)$$

Note that whenever  $V(T) > 0$ ,  $\Pi(x)$  and  $\partial_0 T$  have the same signs. From this equation one may rewrite  $\partial_0 T$  in terms of  $\Pi$  in the following way,

$$\partial_0 T = \frac{\Pi}{\sqrt{\Pi^2 + V^2}}\sqrt{1 + (\nabla T)^2}. \quad (3)$$

In addition to this, we have another useful relation

$$\sqrt{1 - (\partial_0 T)^2 + (\nabla T)^2} = \frac{V}{\sqrt{\Pi^2 + V^2}}\sqrt{1 + (\nabla T)^2}. \quad (4)$$

The Hamiltonian density  $\mathcal{H} = \Pi\partial_0 T - \mathcal{L}$  and the equation of motion of  $\Pi(x)$  are given by

$$\mathcal{H} = \sqrt{\Pi^2 + V^2}\sqrt{1 + (\nabla T)^2}, \quad (5)$$

$$\begin{aligned} \partial_0 \Pi(x) &= -\frac{\delta}{\delta T(x)} \int d^p x \mathcal{H} \\ &= -\frac{V(T)V'(T)}{\sqrt{\Pi^2 + V^2}}\sqrt{1 + (\nabla T)^2} + \partial_i \left( \sqrt{\Pi^2 + V^2} \frac{\partial_i T}{\sqrt{1 + (\nabla T)^2}} \right). \end{aligned} \quad (6)$$

Now we solve the equations of motion. Instead of solving (3) and (6) directly, we make use of the following relation which can be driven from them,

$$\partial_0 \mathcal{H} = \partial_i (\Pi \partial_i T). \quad (7)$$

This, in fact, is  $\nu = 0$  case of  $\partial^\mu T_{\mu\nu} = 0$ , where the energy-momentum tensor is given by

$$T_{\mu\nu} = V \frac{\partial_\mu T \partial_\nu T}{\sqrt{1 + \partial^\lambda T \partial_\lambda T}} - V \eta_{\mu\nu} \sqrt{1 + \partial^\lambda T \partial_\lambda T}. \quad (8)$$

To simplify the problem, we consider first spatially homogeneous case such as  $\partial_i T = 0$ , for all  $i$ . In this case (7) can be written as

$$\frac{d}{dt} \mathcal{H} = 0. \quad (9)$$

This means that  $\Pi^2 + V^2 = \mathcal{H}^2$  is constant during the rolling of the tachyon. Using (2),  $\mathcal{H}^2$  can be written as

$$V^2 \frac{\dot{T}^2}{1 - \dot{T}^2} + V^2 = \mathcal{H}^2. \quad (10)$$

This is reminiscent of the energy-momentum relation

$$m^2 \frac{v^2}{1 - v^2} + m^2 = E^2 \quad (11)$$

of a relativistic particle of mass  $m$  and velocity  $v$ . The crucial difference is that for usual particles masses are constant, but for tachyons  $V(T)$  vanishes rapidly as time goes on.

In this homogeneous field configuration, (10) can be solved formally as

$$t = \int_{T_0}^T \frac{dT}{\sqrt{1 - \frac{V(T)^2}{\mathcal{H}^2}}}, \quad (12)$$

where we assumed that tachyon rolling starts at  $t = 0$  with initial value  $T_0$ . Even though it is not easy to perform the integration explicitly, it can easily be done for the following potential

$$V(T) = e^{-\frac{1}{2}\alpha T} \quad (13)$$

which is known to be an asymptotic of  $V(T)$ . Here,  $\alpha$  is 1 for bosonic strings, and  $\sqrt{2}$  for superstrings. The relevant useful relation is the following,

$$\int \frac{dx}{\sqrt{1 - \frac{1}{b}e^{-ax}}} = \frac{2}{a} \cosh^{-1} \left\{ \sqrt{b} \exp\left(\frac{ax}{2}\right) \right\} + \text{const.} \quad (14)$$

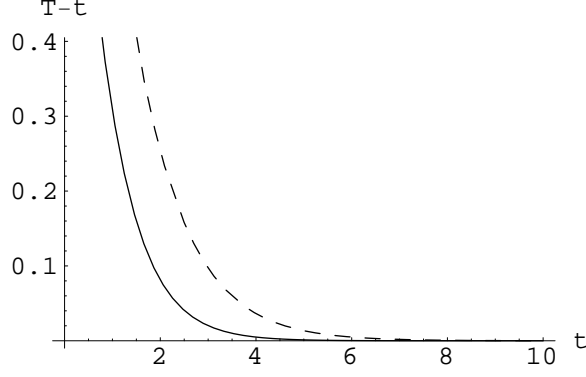


Figure 1: The graph which shows that  $T$  approaches to  $t$  rapidly as  $t \rightarrow \infty$ . The solid line corresponds to superstring case, and the dotted line to bosonic string case. In this plot we assumed that  $T_0 = 0$ .

The explicit solution is given by

$$T = T_0 + \frac{2}{\alpha} \log \cosh \left( \frac{\alpha}{2} t \right) \quad (15)$$

where  $T_0$  is related to  $\mathcal{H}$  by

$$\mathcal{H} = e^{-\frac{1}{2}\alpha T_0}. \quad (16)$$

Using the asymptotic form  $\log \cosh x \sim x$  for large  $x$ , one finds that  $T$  approaches to  $t$  rapidly as  $t \rightarrow \infty$ . In fact it is consistent with the result obtained from Lagrangian formalism[7]. The advantage of our formalism is that it is more convenient when applied to a complex case such as a pair of brane-antibrane which will be considered in the next section.

### 3 Rolling tachyon solution for a brane-antibrane pair

Instead of homogeneous configuration of tachyon field in space, we consider more realistic one by assuming the following special form of solution

$$T = a(|x| - \xi) + \mathcal{T}(t), \quad \text{for } |x| > \xi. \quad (17)$$

Here,  $x$  is one of spatial  $x^i$  coordinate,  $a$  and  $\xi$  are finite positive numbers. This type of configuration corresponds to the tachyon vacuum for large  $|x|$ .

It is known from string theoretic calculations[6] that the potential around the tachyon vacuum looks like (13). It is interesting that this type of ansatz, on the other hand, is valid only if the potential is of type (13). We will prove this fact later. This means that our ansatz is consistent with the general argument of the potential. For  $|x| < \xi$ , the ansatz may not be valid, and may need a cross term of functions of  $x$  and of  $t$ . This term may be related to the possible modification of the potential (13) for small  $T$ . Physical importance the ansatz (17), according to the interpretation of [8], is that it corresponds to the tachyon configuration of a brane-antibrane pair.

To solve the equation of motion for the given form (17), we start from the energy-momentum conservation relation (7),

$$\partial_0 \mathcal{H} = \pm a \frac{\partial \Pi}{\partial x}, \quad (18)$$

where  $\pm$  corresponds to the sign of  $x$ . This can easily be solved when one make use of

$$\frac{\partial}{\partial x} \left( \frac{\Pi}{V} \right) = 0, \quad (19)$$

which is a result of (2). Then (18) reduces to

$$\partial_0 \log \left( \mathcal{H} V^{-\frac{a^2}{1+a^2}} \right) = 0. \quad (20)$$

By using the fact that

$$\mathcal{H} = \frac{b^2}{\sqrt{b^2 - \dot{T}^2}} V, \quad (21)$$

where  $b = \sqrt{1 + a^2}$ , one may prove that (20) reduces to

$$\frac{\ddot{T}(t)}{b^2 - \dot{T}(t)} = -\frac{1}{b^2} \frac{V'(T)}{V(T)}. \quad (22)$$

The left hand side of this equation is independent of  $x$ , while the right hand side of it depends on  $T = a|x - \xi| + \mathcal{T}(t)$ . This means that the only possibility is

$$\frac{V'(T)}{V(T)} = -\frac{\alpha}{2} = \text{const.} \quad (23)$$

This shows that the ansatz (17) is meaningful only for potential of type (13). When one use (23) in (22), the resulting equation can easily solved to give

$$\mathcal{T} = \mathcal{T}_0 + \frac{2b^2}{\alpha} \log \cosh \left\{ \frac{\alpha}{2b}(t - t_0) \right\}, \quad (24)$$

where  $t_0$  and  $\mathcal{T}_0$  are integration constants. Note that as  $t \rightarrow \infty$ ,  $T \rightarrow bt$ .

By comparing the result to the homogeneous case (15), it is clear that the decay mode of the rolling tachyon of a brane-antibrane pair is identical to that of an unstable brane. The only difference is the characteristic decay time which is modified by the factor  $b$ .

## Acknowledgement

This work is supported by the Basic Science Promotion Program of the Chungbuk National University, BSRI-01-S01.

## References

- [1] A. Sen, JHEP **9808**, (1998) 012 [hep-th/9805170](#); JHEP **9912**, (1998) 027 [hep-th/9805170](#); JHEP **0204**, (2002) 048 [hep-th/0203211](#); JHEP **0207**, (2002) 065 [hep-th/0203265](#); [hep-th/0207105](#).
- [2] A. Sen, [hep-th/9904207](#); M. Gutperle and A. Strominger, JHEP **0204**, (2002) 018 [hep-th/0202210](#); J. A. Minahan, JHEP **0207**, (2002) 030 [hep-th/0205098](#); K. Hashimoto, JHEP **0207**, (2002) 035 [hep-th/0204203](#);
- [3] Lev Kofman and Andrei Linde, [hep-th/0205121](#); Jian-gang Hao and Xin-zhou Li, [hep-th/0209041](#); M. Sami, [hep-th/0205146](#); H. B. Benaoum, [hep-th/0205140](#); M. Sami, Pravabati Chingangbam, and Tabish Qureshi, [hep-th/0205179](#); Malcolm Fairbairn and Michel H. G. Tytgat, [hep-th/0204070](#); Alexander Feinstein, [hep-th/0204140](#); Gary Shiu and Ira Wassermann, [hep-th/0205003](#); Andrei Frolov,

Lev Kofman, and Alexei Starobinsky, [hep-th/0204187](#); Debajyoti Choudhury, Debashis Ghoshal, Dileep P. Jatkar, and Sudhakar Panda, [hep-th/0204204](#); T. Padmanabhan, [hep-th/0204150](#).

[4] A. Sen, [hep-th/0209122](#)

[5] G. W. Gibbons, K. Hori and P. Yi, Nucl. Phys. **B 596**, (2001) 136, [hep-th/0009061](#); G. W. Gibbons, K. Hashimoto and P. Yi, [hep-th/0209034](#).

[6] A. Sen, [hep-th/0204143](#), JHEP **9808**, (1998) 010 [hep-th/9805019](#); JHEP **9808**, (1998) 023 [hep-th/9808141](#); JHEP **9812**, (1998) 021 [hep-th/9812031](#); *Int. J. Mod. Phys. A* **14**(1999)4061 [hep-th/9904207](#) D. Kutasov, M. Marino and G. W. Moore, [hep-th/0009148](#); [hep-th/0010108](#); B. Zwiebach, JHEP **0009**, (2000) 028 [hep-th/0008227](#); J. A. Minahan and B. Zwiebach, JHEP **0103**, (2001) 038 [hep-th/0009246](#).

[7] Gary Felder, Lev Kofman, and Alexei Starobinsky, [hep-th/0208019](#).

[8] Koji HASHIMOTO and Norisuke SAKAI, [hep-th/0209232](#).